Journal of Fluids and Structures (1999) **13,** 161–164
Article No.: jfls. 1998.0198.
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LETTER TO THE EDITOR

NONLINEAR VIBRATIONS OF CYLINDRICAL SHELLS — LOGICAL RATIONALE

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THE WRITER was a reviewer of the paper by Amabili *et al.* (1998) and was provided with an advance copy of the recent letter by Dowell (1998). Dean Dowell and I have argued for our respective points of view several times in the past [see references in Dowell (1998)].

Part of my Engineering Consulting practice deals with Expert Witness work: attorneys present arguments grounded in *logic*, not necessarily with equations and mathematics. Accordingly, I have tried to present the logical flow of ideas (as I understand them) which undergird the nonlinear vibrations of cylindrical shells.

Timoshenko's "Plates & Shells" (1959) shows that (for rectangular plates subjected to lateral pressure) nonlinear effects become important when the lateral deflection is approximately one plate thickness, h. Note that the conditions of in-plane edge restraint are important here.

When Chu & Herrmann (1956) reported on the "Non-Linear Vibration of Plates", they found that nonlinear effects became important when the lateral vibration amplitude was approximately one plate thickness. They further demonstrated that the "aspect ratio" was important and that a rectangular plate with edges in the ratio 4 : 1 behaved the same as one with edges 1 : 4. Thus, the results were "symmetric" with respect to the aspect ratio; note that the calculations were performed for zero in-plane displacement at the edges.

Chu (1961) then extended his work to include cylindrical shells, and he concluded that (a) the cylinder vibrations were "always of the hardening type" and (b) they were "strongly nonlinear" (as had been found for the plates). He also reported that (like the previous plate vibrations) the shell vibrations were "symmetric" with respect to "aspect ratio". Note that in the circumferential direction, the circular structure attaches back onto itself; consequently, the in-plane constraint is very different for a closed shell than it is for a flat plate.

About that time, a colleague of mine prompted me to conduct an experiment (Evensen 1963) which showed (a) the nonlinearity was of the "softening type" and (b) the vibrations were only "weakly nonlinear". These experimental results led to a re-examination of the theory and the finding that a periodicity condition had been neglected in the analysis. Discussions with friends and the example of a Russian paper on "dynamic buckling" led the writer to assume a primary mode $\cos(n\theta)\,\sin(\pi x/L)$ and the "now infamous" axisymmetric mode $\sin^2(\pi x/L)$.

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In discussing the flexural vibrations of rings, Rayleigh (1945) shows that they are often very "nearly inextensional". The bending stiffness (EI) is proportional to the thickness cubed (h^3), whereas the stretching thickness (Eh) is proportional to the thickness directly. Consequently, thin rings (and/or shells) deform much more readily in bending than they do in stretching. Apparently, this fact is the reason texts, e.g. Flügge (1973), emphasize the 'membrane theory" when considering the primary load-carrying mechanism of shells. The "bending theory" often relates more to localized deformations and localized stresses.

By using the axisymmetric "sine-squared" contraction term, one is satisfying the "membrane theory" and at the same time violating the "bending theory". This particular axisymmetric mode allows the shell to vibrate nearly inextensionally, but it faills to satisfy the (bending) boundary condition of "simply supported" ends. Thus, using the sine-squared axisymmetric mode avoids circumferential stretching (as much as possible) and involves low energy localized bending (at the ends of the shell).

It was roughly at this point that of the writter's advisors (T.K. Caughey and Y.C. Fung) independently suggested that he concentrate his efforts on rings. Nonlinear flexural vibrations of rings were studied both theoretically and experimentally, including the effects of extensionality, tangential inertia, shear deformation, additional nonlinear effects, the companion mode, etc., (Evensen 1964). The main ideas are summarized in NASA TR-227 (Evensen, 1965), but the nitty-gritty details are found in the thesis itself.

Regarding the cylindrical shell problem, one can argue that "if the shell is long enough, (length/radius tending to infinity) it should behave much like a ring". Furthermore "if the shell is short enough (length/radius tending to zero), then it should behave much like a rectangular plate". That is, "if the equations developed for a cylindrical shell are to be deemed 'accurate', they should be able to achieve the 'limiting case' conditions of L/R tending to infinity and L/R tending to zero". Note again here that the in-plane boundary conditions are important, particularly as (L/R) tends to zero.

These limiting conditions were discussed by Dowell & Ventres (1968) and they serve as important "check points" in the development of the theory. It can be noted here that Chu's original paper on shells (Chu 1961) argued that the results were "symmetric" with respect to the aspect ratio. But the cylinder is curved in only one direction, not two; hence the results cannot be symmetric. Not unexpectedly, Chu's results (1961) fail the limiting value "check point" tests.

The question of simply-supported boundary conditions was dealt with by Ginsberg (1973) and by Chen & Babcock (1975). Both of these papers illustrate the need to include second-harmonic terms in space; e.g., $\cos(2n\theta)$, $\sin(2n\theta)$. A review of the writer's experimental data on rings (Evensen 1964) suggests the presence of such second-harmonic terms in the tests.

Many years ago, Arnold & Warburton (1949) showed that the vibration of cylindrical shells involved (a) bending effects, and (b) stretching effects. High circumferential mode numbers, n, contributed primarily to the bending, and low modal numbers determined the stretching. In the nonlinear case at hand, high axial model numbers, m, are influenced by the simply-supported boundary condition, and low axial mode numbers are relatively insensitive to the simply-supported boundary condition.

On the basis of strain energy considerations, the writer argued (Evensen 1974) that the simply supported boundary condition was relatively unimportant for "long shells" (length/radius tending to become large). Furthermore, in the "membrane limit" (as thickness/radius tends to zero) the moment-free boundary condition disappears entirely. Thus, it can be argued that for many situations the moment-free boundary condition is unimportant and can be neglected.

The question then arises: *Can one set up an experiment to demonstrate these effects, many of which are predicted by theory?* "The nonlinear effects are minuscule — you'll not get anything from experiments", says one researcher. "But what if we apply compressive loads to the shell?", says another. As buckling is approached (and the linear vibration frequency approaches zero) the nonlinear effects can be made larger and larger. One can achieve "strongly nonlinear" behaviour, depending upon the mode involved, etc.

Basically, by vibrating a pre-loaded shell (e.g., with external pressure applied), the linear frequency becomes smaller, the nonlinear effects become relatively more pronounced, and the experimenter hopefully can differentiate between modes which exhibit nonlinear hardening, nonlinear softening, simply supported boundary-condition effects, etc. The writer argued for such tests previously (Evensen 1974), and perhaps their time is finally drawing nigh.

A procedure of this kind was performed by Burgreen (1951), using a column as the test article. As the axial load, P, becomes larger (approaching the critical value of P) the nonlinear effects become stronger. To perform an analogous test on a shell, one has to be careful not to get catastrophic buckling and destroy the test specimen. If one tests a long shell (L/R fairly large, made of Mylar, for example), he may be able to "buckle the shell" without damaging it, particularly, if he subjects it to a relatively benign loading such as external pressure. Loading the shell in axial compression will also work, but it might be more difficult to accomplish experimentally.

Related work on nonlinear vibrations of shells includes Matsuzaki & Kobayshi's (1970) work, limit-cycle travelling-wave flutter (Evensen & Olson 1967), conical shells, and nonlinear finite-element studies. Theoretical papers outnumber the experimental papers on these subjects by perhaps 5 or 10 to one.

In 1963, the time was ripe for simple, appropriate experiments on cylindrical shells. Clearly, experiments on compressively loaded shells are needed today; I have tried to present the rationale for them here.

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